Deterministic Skill Scores

Eadaoin Doddy
eadaoin.doddy@ucdconnect.ie

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Energy Institute
University College Dublin

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1 Introduction

In order to assess the skill of forecasts it is necessary to regularly compare the forecast to observations. This quantifies their skill and assess the situations when the system performs well or not. Gensler et al. [2016] describes common deterministic error scores for power forecasting which are similar to those used in weather forecasting. The following report will describe some deterministic skill scores and in which situation they are most suitable. Also provided is a simple example of the scores in python (all of which require the `NumPy` package).

The score definitions that follow are defined by WMO [2015] where:

$$
\begin{aligned}
x_f &= \text{the forecast value of the parameter in question} \\
x_v &= \text{the corresponding verifying value} \\
x_c &= \text{the climatological value of the parameter} \\
n &= \text{the number of grid points or observations in the verification area} \\
M_{f,c} &= \text{the mean value over the verification area of the forecast anomalies from climate} \\
M_{v,c} &= \text{the mean value over the verification area of the analysed anomalies from climate} \\
\vec{V}_f &= \text{the forecast wind vector} \\
\vec{V}_v &= \text{the corresponding verifying value}
\end{aligned}
$$

The weights w_i applied at each grid point or observation location are defined as:
Verification against analyses: $w_i = cos\phi_i$, cosine of latitude at grid point i.
Verification against observations: $w_i = 1/n$, all observations have equal weight.

2 Error

A simple forecasting error is defined by subtracting the observed from the predicted. The predicted values may be forecast data or data from a reanalysis model.

$$
\begin{aligned}
\text{observed} &= \text{predicted} - \text{error} \\
\text{error} &= \text{predicted} - \text{observed}
\end{aligned}
$$

3 Bias

The bias shows if a model overestimates or underestimates a forecast. It's an average of all single error values. Bias is also known as Mean Error (ME).

$$ME = \sum_{i=1}^{n} w_i (x_f - x_v)_i \tag{1}$$

Positive and negative errors cancel each other out in this score, therefore it provides only an average summary of whether the system overestimates or underestimates, implying a systematic error. The ideal value is zero. Bias is not a measure of the forecasting quality, but a low bias is desirable and related to a low error. If the ME is independent of the forecast the error is called an unconditional bias. If the ME depends on the forecast itself then the error is called conditional bias.

The python code is:

```
ME = np.mean(forecast-observed)
```

4 Mean Absolute Error

The mean absolute error (MAE) sums up the absolute error of each forecast. It is used to determine the overall minimum difference in error values or to find the proportional weighting of errors. It is a linear absolute error measure.

$$MAE = \sum_{i=1}^{n} w_i |x_f - x_v|_i \tag{2}$$

The python code is:

```
MAE = np.mean(abs(forecast-observed))
```

Gensler et al. [2016] describes how the MAE is most suitable for electricity trading as the monetary consequences (cost/reward functions) have a direct relationship to the error of the forecast.

5 Mean Square Error

The mean square error (MSE) factors in the errors quadratically. Therefore, high errors are penalized more, while low errors have lower influence on the overall score.

$$MSE = \sum_{i=1}^{n} w_i (x_f - x_v)_i^2 \tag{3}$$

This score is often used for optimisation purposes during forecast model training. If a forecast has to avoid extreme errors, the MSE score is appropriate.

6 Root Mean Square Error

The most common accuracy measure is the root mean square error (RMSE) which is a measure of the distance between the forecast and the observation. Lower values of RMSE are better.

$$RMSE = \sqrt{\sum_{i=1}^{n} w_i (x_f - x_v)_i^2} \tag{4}$$

As the square-root of the MSE is computed, the value is represented in the original physical unit, making it easier to relate to a forecast value. The RMSE penalises large errors more than the non-quadratic MAE and therefore takes higher numerical values.

The python code is:

```
RMSE = np.sqrt(np.mean((forecast-observed)**2))
```

Gensler et al. [2016] describes how the RMSE should be used instead of MSE when presenting results, as the error units are better understandable. Gensler et al. [2016] also describes the RMSE as the most suitable score for grid operators and other grid stability orientated market participants, as the extreme errors are reflected more appropriately.

7 Anomaly Correlation Coefficient

Correlations between forecasts and observations may have too high correlations due to seasonal variations therefore the anomaly correlation coefficient (ACC) is used. It removes the climate average from both forecast and observations and verifies the anomalies.

$$ACC = \frac{\sum_{i=1}^{n} w_i (x_f - x_c - M_{f,c})_i (x_v - x_c - M_{v,c})_i}{(\sum_{i=1}^{n} w_i (x_f - x_c - M_{f,c})_i^2)^{1/2} (\sum_{i=1}^{n} w_i (x_v - x_c - M_{v,c})_i^2)^{1/2}} \tag{5}$$

Increasing numerical values indicate increasing "success". An ACC=60% corresponds to the range up to which there is synoptic skill for the largest weather patterns. An ACC= 50% corresponds to forecasts for which the error is the same as for a forecast based on a climatological average.

The python code is:

```
def ACC(FC,OBS,CL):
    top = np.mean((FC-CL)*(OBS-CL))
    bottom = np.sqrt(np.mean((FC-CL)**2)*np.mean((OBS-CL)**2))
    ACC = top/bottom
    return ACC
```

where FC are the forecast values, OBS are the observed values and CL are the climate values.

8 RMS Vector Wind Error

The RMS vector wind error is a mandatory score for wind [WMO, 2015], along with the mean error of wind speed.

$$\mathrm{rms} = \sqrt{\sum_{i=1}^{n} w_i (\vec{V}_f - \vec{V}_v)_i^2} \tag{6}$$

Bibliography

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